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Advanced Strength and Applied Elasticity (4th Edition)

Chapter 4, Problem 33P Bookmark Show all steps: ON

Step-by-step solution

Step 1 of 33

Consider a rotating hub and shaft assembly that is subjected to bending moment, axial thrust, bidirectional torque, and a uniform shrink fit pressure so that the following stress levels occur at an outer critical point of the shaft.

$$\begin{bmatrix} 1000 & 300 & 0 \\ 300 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -800 & -100 & 0 \\ -100 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 2 of 33

Calculate the alternating stress σ_{xa} in the x-direction.

$$\sigma_{xa} = \frac{1}{2}\left(\sigma_{x\max} - \sigma_{x\min}\right)$$

Substitute 1000 MPa for $\sigma_{x\max}$ and -800 MPa for $\sigma_{x\min}$.

$$\begin{aligned} \sigma_{xa} &= \frac{1}{2}\left(1000\text{ MPa} - (-800\text{ MPa})\right) \\ &= 900\text{ MPa} \end{aligned}$$

Step 3 of 33

Calculate the mean stress σ_{xm} in the x-direction.

$$\sigma_{xm} = \frac{1}{2}\left(\sigma_{x\max} + \sigma_{x\min}\right)$$

Substitute 1000 MPa for $\sigma_{x\max}$ and -800 MPa for $\sigma_{x\min}$.

$$\begin{aligned} \sigma_{xm} &= \frac{1}{2}\left(1000\text{ MPa} + (-800\text{ MPa})\right) \\ &= 100\text{ MPa} \end{aligned}$$

Step 4 of 33

Calculate the alternating shear stress τ_{xya}.

$$\tau_{xya} = \frac{1}{2}\left(\tau_{xy\max} - \tau_{xy\min}\right)$$

Substitute 300 MPa for $\tau_{xy\max}$ and -100 MPa for $\tau_{xy\min}$.

$$\begin{aligned} \tau_{xya} &= \frac{1}{2}\left(300\text{ MPa} - (-100\text{ MPa})\right) \\ &= 200\text{ MPa} \end{aligned}$$

Step 5 of 33

Calculate the mean shear stress τ_{xym}.

$$\tau_{xym} = \frac{1}{2}\left(\tau_{xy\max} + \tau_{xy\min}\right)$$

Substitute 300 MPa for $\tau_{xy\max}$ and -100 MPa for $\tau_{xy\min}$.

$$\begin{aligned} \tau_{xym} &= \frac{1}{2}\left(300\text{ MPa} + (-100\text{ MPa})\right) \\ &= 100\text{ MPa} \end{aligned}$$

Step 6 of 33

Now apply the maximum shear stress theory to the shaft and determine the principal alternating stresses σ_{1a} and σ_{2a}.

$$\sigma_{1a,2a} = \frac{\sigma_{xa} + \sigma_{ya}}{2} \pm \sqrt{\left(\frac{\sigma_{xa} + \sigma_{ya}}{2}\right)^2 + \left(\tau_{xya}\right)^2}$$

Step 7 of 33

Substitute 900 MPa for σ_{xa} and 200 MPa for τ_{xya}.

$$\begin{aligned} \sigma_{1a,2a} &= \frac{900\text{ MPa}}{2} \pm \sqrt{\left(\frac{900\text{ MPa}}{2}\right)^2 + (200\text{ MPa})^2} \\ \sigma_{1a} &= 942.44\text{ MPa} \\ \sigma_{2a} &= -42.44\text{ MPa} \end{aligned}$$

Step 8 of 33

Calculate the equivalent alternating stress σ_{ea}.

$$\sigma_{ea} = \sigma_{1a} - \sigma_{2a}$$

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Substitute 942.44 MPa for σ_{1a} and -42.44 MPa for σ_{2a}.

$$\begin{aligned} \sigma_{ea} &= 942.44\text{ MPa} - (-42.44\text{ MPa}) \\ &= 984.88\text{ MPa} \end{aligned}$$

Step 10 of 33

Now apply the maximum shear stress theory to the shaft and determine the principal mean stresses σ_{1m} and σ_{2m}.

$$\sigma_{1m,2m} = \frac{\sigma_{xm} + \sigma_{ym}}{2} \pm \sqrt{\left(\frac{\sigma_{xm} + \sigma_{ym}}{2}\right)^2 + \left(\tau_{xym}\right)^2}$$

Step 11 of 33

Substitute 100 MPa for σ_{xm} and 100 MPa for τ_{xym}.

$$\begin{aligned} \sigma_{1m,2m} &= \frac{100\text{ MPa}}{2} \pm \sqrt{\left(\frac{100\text{ MPa}}{2}\right)^2 + (100\text{ MPa})^2} \\ \sigma_{1m} &= 161.80\text{ MPa} \\ \sigma_{2m} &= -61.80\text{ MPa} \end{aligned}$$

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Step 13 of 33

Calculate the equivalent mean stress σ_{em}.

$$\sigma_{em} = \sigma_{1m} - \sigma_{2m}$$

Step 14 of 33

Substitute 161.80 MPa for σ_{1m} and -61.80 MPa for σ_{2m}.

$$\begin{aligned} \sigma_{em} &= 161.80\text{ MPa} - (-61.80\text{ MPa}) \\ &= 223.60\text{ MPa} \end{aligned}$$

Step 15 of 33

Apply the Modified Goodman criterion to determine the completely reversed stress σ_{cr}.

$$\begin{aligned} \frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_u} &= 1 \\ \frac{\sigma_a}{\sigma_{cr}} &= 1 - \frac{\sigma_m}{\sigma_u} \\ \sigma_a &= \sigma_{cr}\left(1 - \frac{\sigma_m}{\sigma_u}\right) \\ \sigma_{cr} &= \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_u}\right)} \end{aligned}$$

Step 16 of 33

Substitute 984.88 MPa for σ_a, 223.60 MPa for σ_m, and 2400 MPa for σ_u.

$$\begin{aligned} \sigma_{cr} &= \frac{984.88\text{ MPa}}{\left(1 - \frac{223.60\text{ MPa}}{2400\text{ MPa}}\right)} \\ &= 1086\text{ MPa} \end{aligned}$$

Step 17 of 33

Calculate the constant *b*.

$$b = \frac{\ln\left(\sigma_f / \sigma_e\right)}{\ln\left(N_f / N_e\right)}$$

Here, σ_f is the fracture stress, σ_e is the fatigue strength, N_f is the fracture cycles, and N_e is the fatigue cycles.

Using the fracture stress and fatigue strength table for brittle steels (because the static tensile ultimate stress 2400 MPa is greater than 1750 MPa), subsitute 0.9 σ_u for σ_f, 10^3 for N_f, $\frac{1}{3}K\sigma_u$ for σ_e and 10^8 for N_e.

$$b = \frac{\ln\left(0.9\sigma_u \Big/ \left(\frac{1}{3}K\sigma_u\right)\right)}{\ln\left(10^3 / 10^8\right)}$$

Substitute 2400 MPa for σ_u and 1 for *K*.

$$\begin{aligned} b &= \frac{\ln\left(0.9(2400\text{ MPa}) \Big/ \left(\frac{1}{3}(1)(2400\text{ MPa})\right)\right)}{\ln\left(10^3 / 10^8\right)} \\ &= -0.0863 \end{aligned}$$

Step 18 of 33

Solve for the fatigue life N_{cr} based on the Modified Goodman criterion.

$$N_{cr} = N_f\left(\frac{\sigma_{cr}}{\sigma_f}\right)^{1/b}$$

Substitute 10^3 for N_f, 1086 MPa for σ_{cr}, 0.9 σ_u for σ_f, -0.0863 for *b*, and 2400 MPa for σ_u.

$$\begin{aligned} N_{cr} &= 10^3\left(\frac{1086\text{ MPa}}{0.9\sigma_u}\right)^{1/-0.0863} \\ &= 10^3\left(\frac{1086\text{ MPa}}{0.9(2400\text{ MPa})}\right)^{1/-0.0863} \\ &= 2.89\left(10^6\right)\text{ cycles} \end{aligned}$$

Therefore the fatigue life based on the Modified Goodman criterion is $\boxed{2.89\left(10^6\right)\text{ cycles}}$

Step 19 of 33

Apply the Soderberg criterion to determine the completely reversed stress σ_{cr}.

$$\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_{yp}} = 1$$

Here σ_{cr} is the completely reversed stress and σ_{yp} is the static tensile yield stress.

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$$\begin{aligned} \frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_{yp}} &= 1 \\ \frac{\sigma_a}{\sigma_{cr}} &= 1 - \frac{\sigma_m}{\sigma_{yp}} \\ \sigma_a &= \sigma_{cr}\left(1 - \frac{\sigma_m}{\sigma_{yp}}\right) \\ \sigma_{cr} &= \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_{yp}}\right)} \end{aligned}$$

Step 21 of 33

Substitute 984.88 MPa for σ_a, 223.60 MPa for σ_m, and 1600 MPa for σ_{yp}.

$$\begin{aligned} \sigma_{cr} &= \frac{984.88\text{ MPa}}{\left(1 - \frac{223.60\text{ MPa}}{1600\text{ MPa}}\right)} \\ &= 1145\text{ MPa} \end{aligned}$$

Step 22 of 33

Calculate the constant *b*.

$$b = \frac{\ln\left(\sigma_f / \sigma_e\right)}{\ln\left(N_f / N_e\right)}$$

Here, σ_f is the fracture stress, σ_e is the fatigue strength, N_f is the fracture cycles, and N_e is the fatigue cycles.

Using the fracture stress and fatigue strength table for brittle steels (because the static tensile ultimate stress 2400 MPa is greater than 1750 MPa), subsitute 0.9 σ_u for σ_f, 10^3 for N_f, $\frac{1}{3}K\sigma_u$ for σ_e and 10^8 for N_e.

$$b = \frac{\ln\left(0.9\sigma_u \Big/ \left(\frac{1}{3}K\sigma_u\right)\right)}{\ln\left(10^3 / 10^8\right)}$$

Substitute 2400 MPa for σ_u and 1 for *K*.

$$\begin{aligned} b &= \frac{\ln\left(0.9(2400\text{ MPa}) \Big/ \left(\frac{1}{3}(1)(2400\text{ MPa})\right)\right)}{\ln\left(10^3 / 10^8\right)} \\ &= -0.0863 \end{aligned}$$

Step 23 of 33

Solve for the fatigue life N_{cr} based on the Soderberg criterion.

$$N_{cr} = N_f\left(\frac{\sigma_{cr}}{\sigma_f}\right)^{1/b}$$

Substitute 10^3 for N_f, 1145 MPa for σ_{cr}, 0.9 σ_u for σ_f, -0.0863 for *b*, and 2400 MPa for σ_u.

$$\begin{aligned} N_{cr} &= 10^3\left(\frac{1145\text{ MPa}}{0.9\sigma_u}\right)^{1/-0.0863} \\ &= 10^3\left(\frac{1145\text{ MPa}}{0.9(2400\text{ MPa})}\right)^{1/-0.0863} \\ &= 1.56\left(10^6\right)\text{ cycles} \end{aligned}$$

Therefore the fatigue life based on the Soderberg criterion is $\boxed{1.56\left(10^6\right)\text{ cycles}}$.

Step 24 of 33

Apply the SAE criterion to determine the completely reversed stress σ_{cr}.

$$\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_f} = 1$$

Here σ_{cr} is the completely reversed stress and σ_{yp} is the static tensile yield stress.

$$\begin{aligned} \frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_f} &= 1 \\ \frac{\sigma_a}{\sigma_{cr}} &= 1 - \frac{\sigma_m}{\sigma_f} \\ \sigma_a &= \sigma_{cr}\left(1 - \frac{\sigma_m}{\sigma_f}\right) \\ \sigma_{cr} &= \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_f}\right)} \end{aligned}$$

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Substitute 984.88 MPa for σ_a, 223.60 MPa for σ_m, and σ_u for σ_f.

$$\sigma_{cr} = \frac{984.88\text{ MPa}}{\left(1 - \frac{223.60\text{ MPa}}{0.9\sigma_u}\right)}$$

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Substitute 2400 MPa for σ_u.

$$\begin{aligned} \sigma_{cr} &= \frac{984.88\text{ MPa}}{\left(1 - \frac{223.60\text{ MPa}}{2400\text{ MPa}}\right)} \\ &= 1086\text{ MPa} \end{aligned}$$

Step 27 of 33

Calculate the constant *b*.

$$b = \frac{\ln\left(\sigma_f / \sigma_e\right)}{\ln\left(N_f / N_e\right)}$$

Here, σ_f is the fracture stress, σ_e is the fatigue strength, N_f is the fracture cycles, and N_e is the fatigue cycles.

Using the fracture stress and fatigue strength table for brittle steels (because the static tensile ultimate stress 2400 MPa is greater than 1750 MPa), subsitute σ_u for σ_f, 1 for N_f, $\frac{1}{3}K\sigma_u$ for σ_e and 10^8 for N_e.

$$b = \frac{\ln\left(\sigma_u \Big/ \left(\frac{1}{3}K\sigma_u\right)\right)}{\ln\left(1 / 10^8\right)}$$

Substitute 2400 MPa for σ_u and 1 for *K*.

$$\begin{aligned} b &= \frac{\ln\left((2400\text{ MPa}) \Big/ \left(\frac{1}{3}(1)(2400\text{ MPa})\right)\right)}{\ln\left(1 / 10^8\right)} \\ &= -0.0596 \end{aligned}$$

Step 28 of 33

Solve for the fatigue life N_{cr} based on the SAE criterion.

$$N_{cr} = N_f\left(\frac{\sigma_{cr}}{\sigma_f}\right)^{1/b}$$

Substitute 1 for N_f, 1086 MPa for σ_{cr}, σ_u for σ_f, -0.05963 for *b*, and 2400 MPa for σ_u.

$$\begin{aligned} N_{cr} &= 1\left(\frac{1086\text{ MPa}}{\sigma_u}\right)^{1/-0.0596} \\ &= \left(\frac{1086\text{ MPa}}{2400\text{ MPa}}\right)^{1/-0.0596} \\ &= 0.60\left(10^6\right)\text{ cycles} \end{aligned}$$

Therefore the fatigue life based on the SAE criterion is $\boxed{0.60\left(10^6\right)\text{ cycles}}$.

Step 29 of 33

Apply the Gerber criterion to determine the completely reversed stress σ_{cr}.

$$\begin{aligned} \frac{\sigma_a}{\sigma_{cr}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 &= 1 \\ \frac{\sigma_a}{\sigma_{cr}} &= 1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2 \\ \sigma_a &= \sigma_{cr}\left(1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right) \\ \sigma_{cr} &= \frac{\sigma_a}{\left(1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right)} \end{aligned}$$

Step 30 of 33

Substitute 984.88 MPa for σ_a, 223.60 MPa for σ_m, and 2400 MPa for σ_u.

$$\begin{aligned} \sigma_{cr} &= \frac{984.88\text{ MPa}}{\left(1 - \left(\frac{223.60\text{ MPa}}{2400\text{ MPa}}\right)^2\right)} \\ &= 994\text{ MPa} \end{aligned}$$

Step 31 of 33

Calculate the constant *b*.

$$b = \frac{\ln\left(\sigma_f / \sigma_e\right)}{\ln\left(N_f / N_e\right)}$$

Here, σ_f is the fracture stress, σ_e is the fatigue strength, N_f is the fracture cycles, and N_e is the fatigue cycles.

Step 32 of 33

Using the fracture stress and fatigue strength table for brittle steels (because the static tensile ultimate stress 2400 MPa is greater than 1750 MPa), subsitute 0.9 σ_u for σ_f, 10^3 for N_f, $\frac{1}{3}K\sigma_u$ for σ_e and 10^8 for N_e.

$$b = \frac{\ln\left(0.9\sigma_u \Big/ \left(\frac{1}{3}K\sigma_u\right)\right)}{\ln\left(10^3 / 10^8\right)}$$

Substitute 2400 MPa for σ_u and 1 for *K*.

$$\begin{aligned} b &= \frac{\ln\left(0.9(2400\text{ MPa}) \Big/ \left(\frac{1}{3}(1)(2400\text{ MPa})\right)\right)}{\ln\left(10^3 / 10^8\right)} \\ &= -0.0863 \end{aligned}$$

Step 33 of 33

Solve for the fatigue life N_{cr} based on the Gerber criterion.

$$N_{cr} = N_f\left(\frac{\sigma_{cr}}{\sigma_f}\right)^{1/b}$$

Substitute 10^3 for N_f, 994 MPa for σ_{cr}, 0.9 σ_u for σ_f, -0.0863 for *b*, and 2400 MPa for σ_u.

$$\begin{aligned} N_{cr} &= 10^3\left(\frac{994\text{ MPa}}{0.9\sigma_u}\right)^{1/-0.0863} \\ &= 10^3\left(\frac{994\text{ MPa}}{0.9(2400\text{ MPa})}\right)^{1/-0.0863} \\ &= 8.05\left(10^6\right)\text{ cycles} \end{aligned}$$

Therefore the fatigue life based on the Gerber criterion is $\boxed{8.05\left(10^6\right)\text{ cycles}}$.

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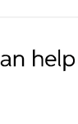

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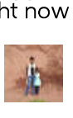

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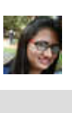

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